$$\phi = V_\infty x + \frac{\Lambda}{4\pi} \ln\left((x-d)^2 + y^2\right)$$

a) $$u = \frac{\partial \phi}{\partial x} = V_\infty + \frac{\Lambda}{2\pi} \frac{x-d}{(x-d)^2 + y^2}$$

$$v = \frac{\partial \phi}{\partial y} = \frac{\Lambda}{2\pi} \frac{y}{(x-d)^2 + y^2}$$

at $x, y = 0, 0$, require $u = 0$

or $$V_\infty - \frac{\Lambda}{2\pi d} = 0$$

$$2\pi V_\infty d = \Lambda \qquad (1)$$

at $x, y = d, \sqrt{Cd}$, require $\frac{v}{u} = \frac{dY}{dx}$, where $\frac{dY}{dx} = \frac{1}{2}\sqrt{\frac{C}{x}}\Big|_{x=d} = \frac{1}{2}\sqrt{\frac{C}{d}}$

or $$\frac{1}{V_\infty} \frac{\Lambda}{2\pi} \frac{\sqrt{Cd}}{Cd} = \frac{1}{2}\sqrt{\frac{C}{d}}$$

$$\boxed{\Lambda = \pi V_\infty C} \qquad (2)$$

Combine (1) & (2) $\rightarrow$ $C = 2d$ $\rightarrow$ $\boxed{d = C/2}$

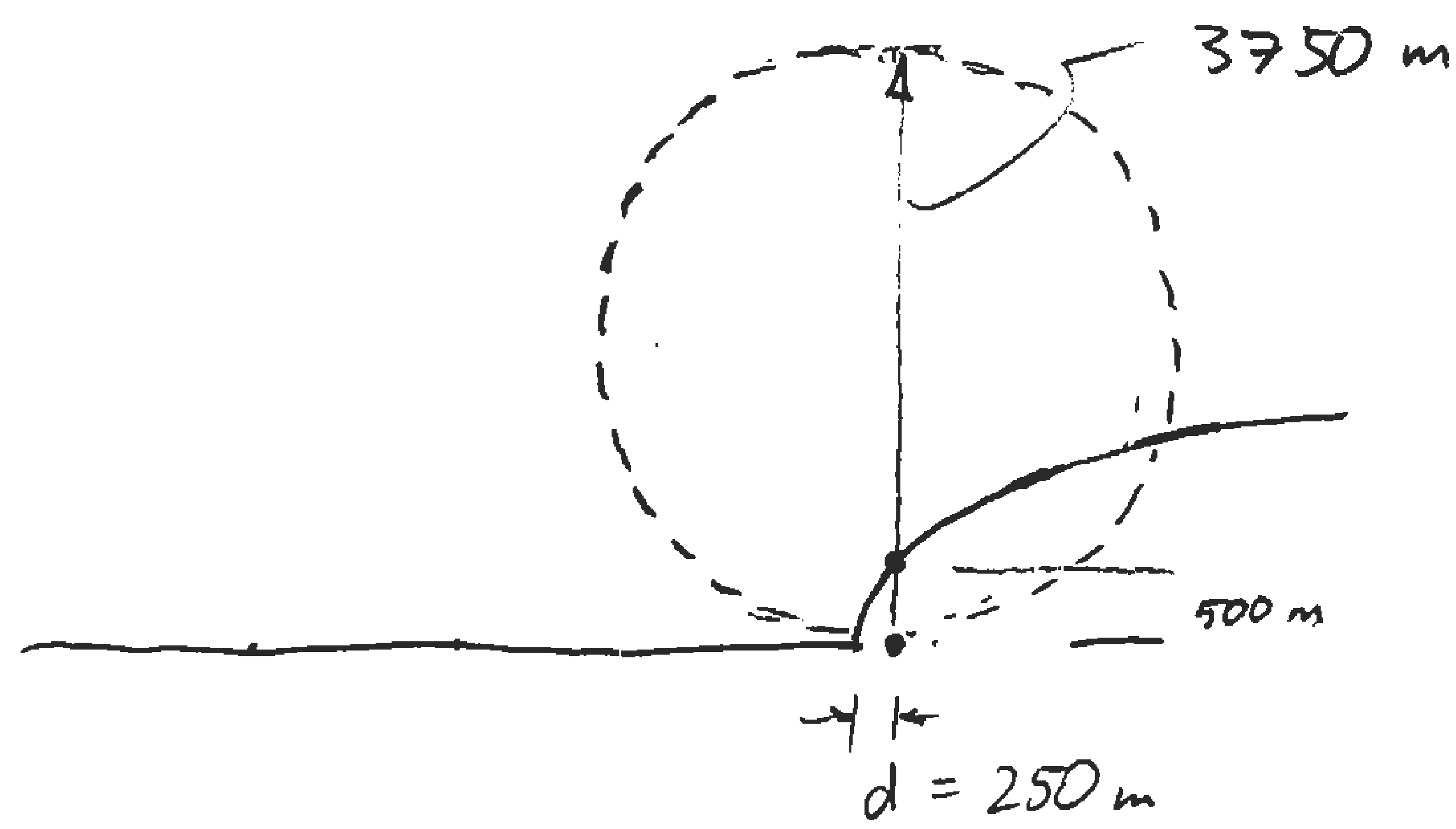
b) For $C = 500\,m$, $V_\infty = 15\,m/s$, $\rightarrow$ $d = 250\,m$, $\Lambda = 7500\pi\ m^2/s$

Maximum radius where $V = 1\,m/s$

or $$V = \frac{\Lambda}{2\pi} \frac{y}{r^2} = \frac{\Lambda}{2\pi} \frac{\sin\theta}{r} = 1\,m/s$$

$$\rightarrow \boxed{r_{max}(\theta) = \frac{\Lambda}{2\pi \cdot 1\,m/s} \sin\theta = 3750\,m \cdot \sin\theta}$$

circle of diameter 3750 m above source.



$$C_p = 1 - \frac{V^2}{V_\infty^2} = 1 - \left(\frac{u}{V_\infty}\right)^2 - \left(\frac{v}{V_\infty}\right)^2$$

a) Source: $u = V_\infty + \frac{\Lambda}{2\pi}\frac{x}{x^2+y^2}$

$v = \frac{\Lambda}{2\pi}\frac{y}{x^2+y^2}$

dominant — much smaller for large x

Along $-x$, $y=0$: $C_p = 1 - \frac{u^2}{V_\infty^2} = 1 - \left(\frac{V_\infty}{V_\infty} - \frac{\Lambda}{2\pi V_\infty}\frac{1}{x}\right)^2 = \boxed{\frac{\Lambda}{\pi V_\infty}\frac{1}{x}} - \left(\frac{\Lambda}{2\pi V_\infty}\right)^2\frac{1}{x^2}$

Along y, $x=0$: $C_p = 1 - \left(\frac{V_\infty}{V_\infty}\right)^2 - \left(\frac{\Lambda}{2\pi V_\infty}\right)^2\frac{1}{y^2} = -\left(\frac{\Lambda}{2\pi V_\infty}\right)^2\frac{1}{y^2}$

b) Vortex: $u = V_\infty + \frac{\Gamma}{2\pi}\frac{y}{x^2+y^2}$

$v = \frac{\Gamma}{2\pi}\frac{-x}{x^2+y^2}$

dominant

Along $-x$, $y=0$: $C_p = 1 - \left(\frac{V_\infty}{V_\infty}\right)^2 - \left(\frac{\Gamma}{2\pi V_\infty}\right)^2\frac{1}{x^2} = -\left(\frac{\Gamma}{2\pi V_\infty}\right)^2\frac{1}{x^2}$

Along y, $x=0$: $C_p = 1 - \left(\frac{u}{V_\infty}\right)^2 = 1 - \left(\frac{V_\infty}{V_\infty} + \frac{\Gamma}{2\pi V_\infty}\frac{1}{y}\right)^2 = \boxed{-\frac{\Gamma}{\pi V_\infty}\frac{1}{y}} - \left(\frac{\Gamma}{2\pi V_\infty}\right)^2\frac{1}{y^2}$

c) Doublet: $\phi = V_\infty x + \frac{\kappa}{2\pi}\frac{x}{x^2+y^2}$

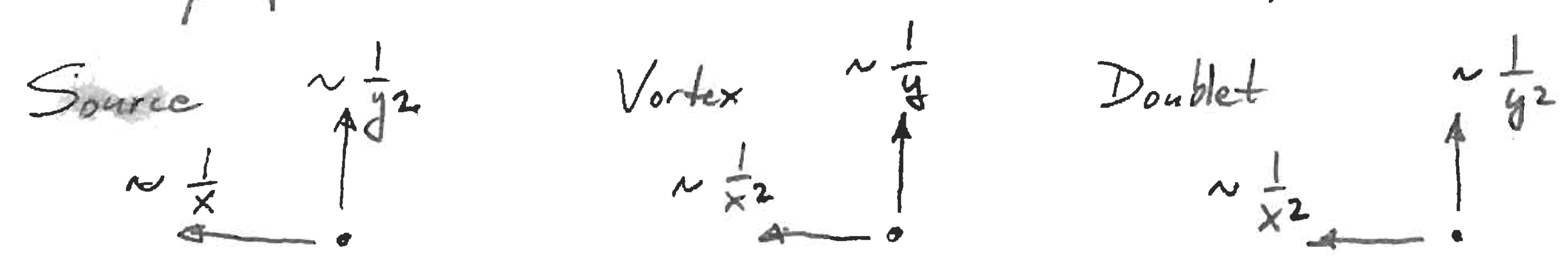
$u = \frac{\partial\phi}{\partial x} = V_\infty + \frac{\kappa}{2\pi}\frac{y^2-x^2}{(x^2+y^2)^2}$

$v = \frac{\partial\phi}{\partial y} = \frac{\kappa}{2\pi}\frac{-2xy}{(x^2+y^2)^2}$

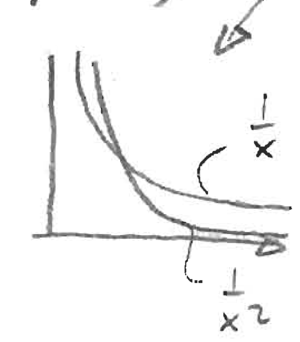
Along $-x$, $y=0$: $C_p = 1 - \left(\frac{u}{V_\infty}\right)^2 = 1 - \left(\frac{V_\infty}{V_\infty} - \frac{\kappa}{2\pi V_\infty}\frac{1}{x^2}\right)^2 = \frac{\kappa}{\pi V_\infty}\frac{1}{x^2} - \left(\frac{\kappa}{2\pi V_\infty}\right)^2\frac{1}{x^4}$

Along y, $x=0$: $C_p = 1 - \left(\frac{u}{V_\infty}\right)^2 = 1 - \left(\frac{V_\infty}{V_\infty} + \frac{\kappa}{2\pi V_\infty}\frac{1}{y^2}\right)^2 = \frac{-\kappa}{\pi V_\infty}\frac{1}{y^2} - \left(\frac{\kappa}{2\pi V_\infty}\right)^2\frac{1}{y^4}$

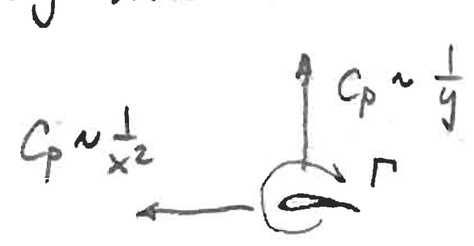
The C_p fields decrease with distance as follows:



Far away the $\frac{1}{x}$ and $\frac{1}{y}$ terms dominate ($\frac{1}{x^2}$ and $\frac{1}{y^2}$ die off much faster)



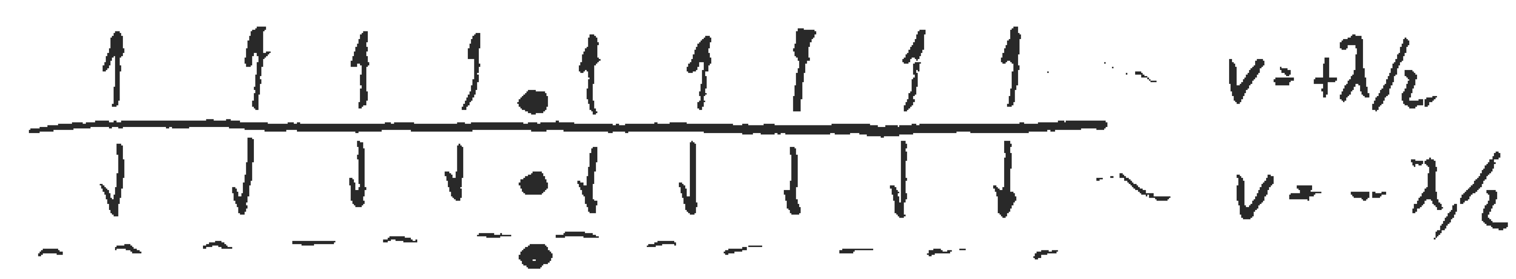
A lifting airfoil has a nonzero Γ, so it looks mostly like a vortex far away. Largest C_p is above & below.



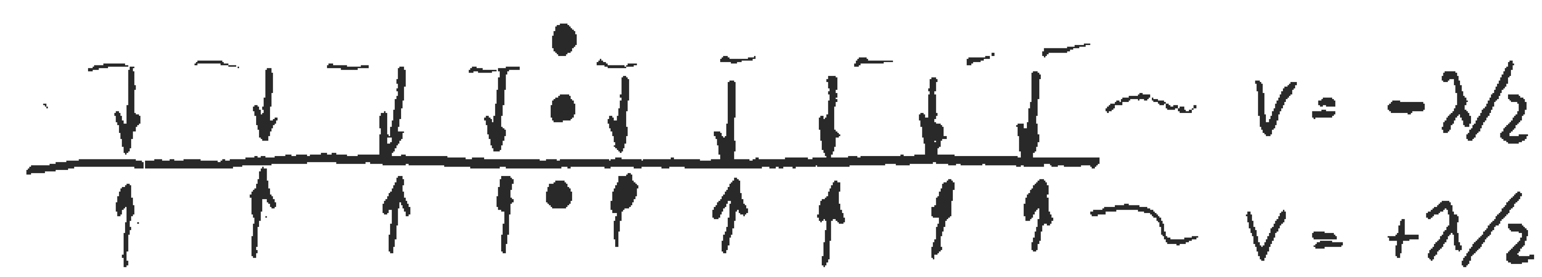
Since the panels are very long, the limiting case $y \to 0$ applies for all 3 points A, B, C. (i.e. h is irrelevant.).

Only vertical velocities are nonzero.

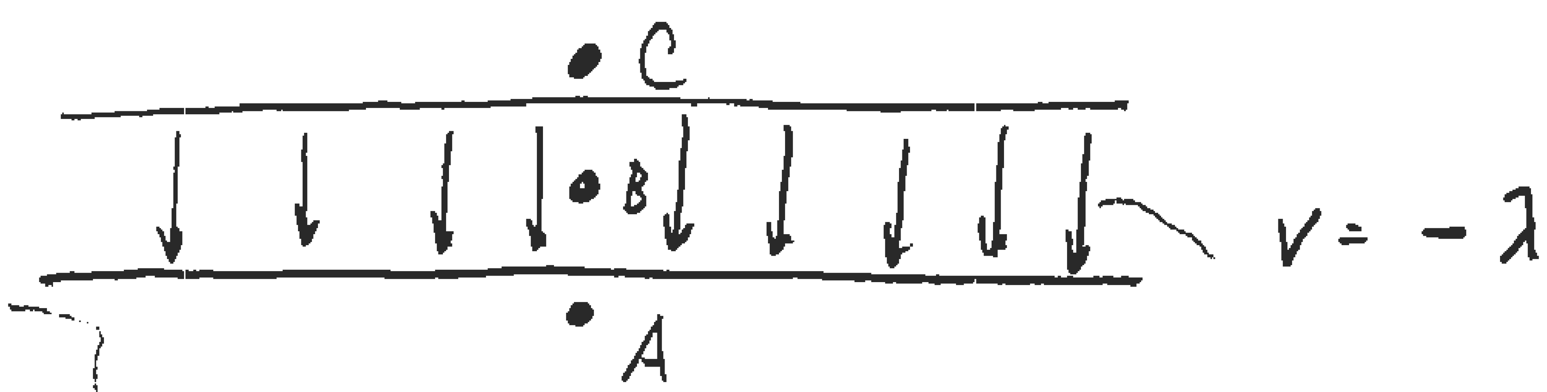
Top panel alone:



Bottom panel alone:



Super impose:



Point A: $v = -\lambda/2 + \lambda/2 = 0$

B: $v = -\lambda/2 - \lambda/2 = -\lambda$

C: $v = \lambda/2 - \lambda/2 = 0$

The velocity field is analogous to the electric field of a capacitor.

M19.

a) This is a transversely isotropic material
So it will require 5 elastic constants

b) Specimen (1) is loaded in the longitudinal fibre direction $\varepsilon_L =$

$$E_L = \frac{\sigma_L}{\varepsilon_L} = \frac{14\times10^3}{(10\times10^{-3})^2} \times \frac{1}{500\times10^{-6}} = 2.8\times10^{11} = 280\,\text{GPa}$$

$$\nu_{LT} = -\frac{\varepsilon_T}{\varepsilon_L} = \frac{-(-120)}{500} = 0.24$$

$$E_T = \frac{\sigma_T}{\varepsilon_T} = \frac{14\times10^3}{(10\times10^{-3})^2} \times \frac{1}{700\times10^{-6}} = 2.0\times10^{11} = 200\,\text{GPa} \Leftarrow$$

$$\nu_{TL} = \frac{-\overset{\text{gauge } c}{\varepsilon_c}}{\underset{\text{gauge } d}{\varepsilon_d}} = \frac{\cancel{14\times10^6}\;-(-125)}{700\,\cancel{\times10^6}} = 0.18 \Leftarrow$$

$$\nu_{TT} = -\frac{\varepsilon_f}{\varepsilon_d} = -\frac{210}{700} = 0.3$$

$$\text{Hence } G_{TT} = \frac{E_{TT}}{2(1+\nu_{TT})} = 77\,\text{GPa} \Leftarrow$$

c) from longitudinal modulus:

$$E_L = V_f E_f + (1 - V_f) E_m$$

$$E_L \cancel{V_f} = V_f (E_f - E_m) + E_m$$

$$\frac{E_L - E_M}{E_f - E_m} = V_f = \frac{280 - 110}{450 - 110} = 0.5$$

for transverse modulus, lower bound estimate

$$E_T = \frac{1}{\frac{V_f}{E_f} + \frac{1 - V_f}{E_M}}$$

$$\frac{E_T V_f}{E_f} + \frac{E_T (1 - V_f)}{E_M} = 1$$

$$E_T E_M V_f + E_T E_f (1 - V_f) = 1$$

$$V_f (E_T E_M - E_T E_F) = - E_T E_f$$

$$V_f = \frac{-\cancel{E_T} E_F}{\cancel{E_T} E_M - \cancel{E_T} E_f} = \frac{-200 \times 450}{200 \times 110 - 200 \times}$$

$$= \frac{E_f}{E_f - E_M} = \frac{450}{450 - 110} = \frac{450}{340} = 1.32$$

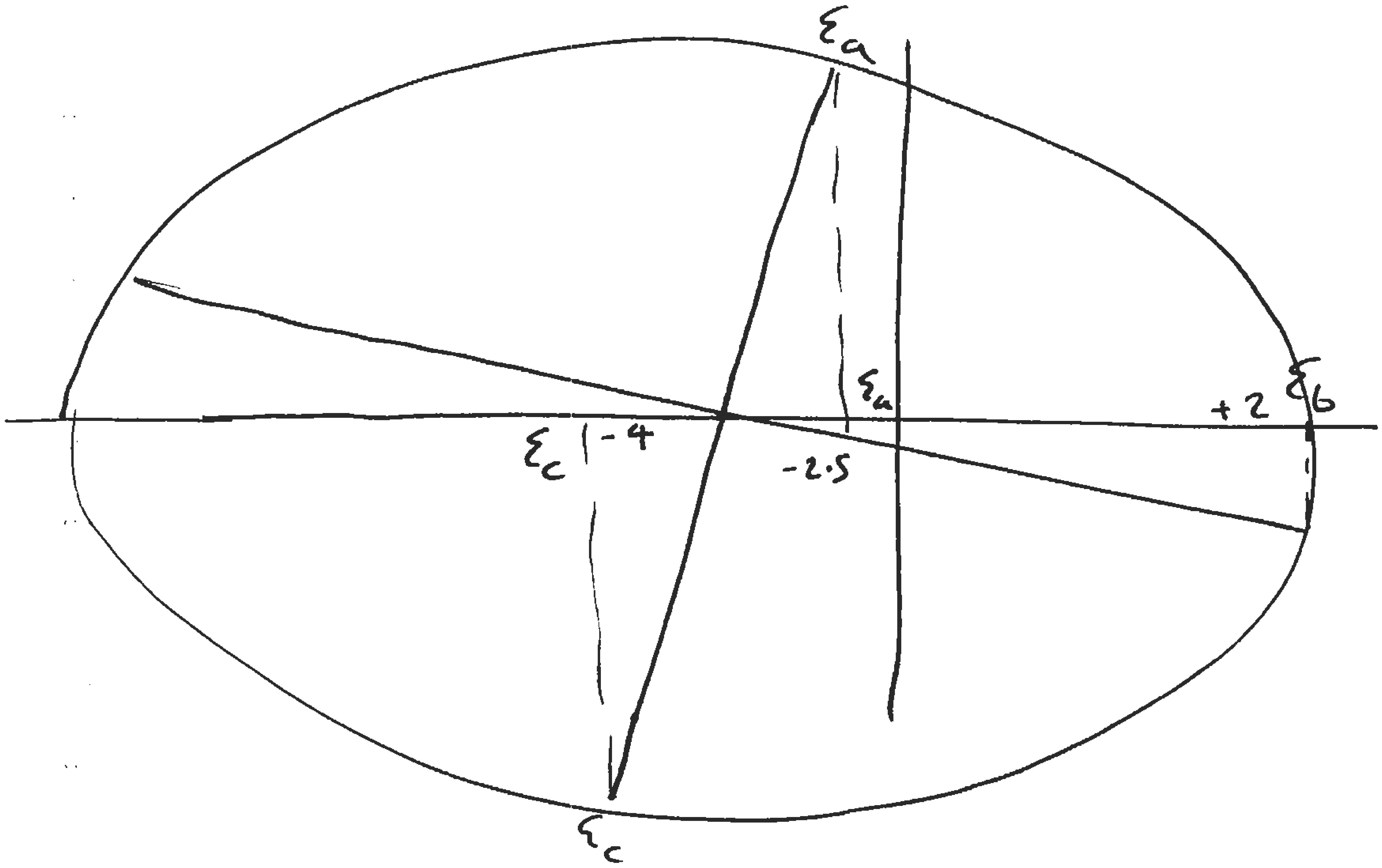
This is greater than 0.5 so not inconsistent!

M20 45° Rosette

$\varepsilon_a = -2.5\ m\varepsilon = 2500\ \mu\varepsilon$

$\varepsilon_b = +2.0\ m\varepsilon = 2000\ \mu\varepsilon$

$\varepsilon_c = -4.0\ m\varepsilon = 4000\ \mu\varepsilon$



center of circle @ $-3.25\ m\varepsilon$

Radius $= \sqrt{(2-(-3.25))^2 + (3.25-2.5)^2} = 5.3\ m\varepsilon$

Principal Strains $= -3.25\ m\varepsilon \pm 5.3\ m\varepsilon$

$\varepsilon_I = +2053\ \mu\varepsilon \qquad \varepsilon_{II} = -8553\ \mu\varepsilon$

a) State of Strain

$\varepsilon_{11} = \varepsilon_a = -2500\ \mu\varepsilon$, $\varepsilon_c = \varepsilon_{22} = -4000\ \mu\varepsilon$, $\varepsilon_{12} = \frac{1}{2}(20-(-32.5))$
$= 2625\ \mu\varepsilon$

$\varepsilon_{\tilde{1}\tilde{1}} = \varepsilon_b = +2000\ \mu\varepsilon$, $\varepsilon_{\tilde{2}\tilde{2}} = -32.5-(52.5) = -8500\ \mu\varepsilon$, $\varepsilon_{\tilde{1}\tilde{2}} = \frac{1}{2}(7.5)$
$= 3750\ \mu\varepsilon$

From Elasticity — Plane Stress

$$\varepsilon_{11} = \frac{\sigma_{11}}{E} - \frac{\nu\sigma_{22}}{E} - \frac{\nu\sigma_{33}}{E}^{=0} \qquad (1)$$

$$\varepsilon_{22} = -\frac{\nu\sigma_{11}}{E} + \frac{\sigma_{22}}{E} - \frac{\nu\sigma_{33}}{E} = 0 \qquad (2)$$

Multiply through by (1) by ν add to (2)

$$\nu\varepsilon_{11} + \varepsilon_{22} = \frac{\sigma_{22}}{E}(1-\nu^2)$$

$$\sigma_{22} = \frac{E(\nu\varepsilon_{11} + \varepsilon_{22})}{(1-\nu^2)} = \frac{70\times10^9(0.33\times(-2500)+(-4000))\times10^{-6}}{(1-(0.33)^2)}$$

$\sigma_c = \sigma_{22} = -380$ MPa ⇐

$\sigma_a = \sigma_{11} = -300$ MPa ⇐ $\left(\frac{E(\nu\varepsilon_{22}+\varepsilon_{11})}{(1-\nu^2)}\right)$

Similarly for $\sigma_b = \sigma_{\bar{1}\bar{1}} = \frac{E(\nu\varepsilon_{\bar{2}\bar{2}} + \varepsilon_{\bar{1}\bar{1}})}{(1-\nu^2)} = -63$ MPa ⇐

Principal stresses from principal strains

$$\sigma_I = \frac{E(\varepsilon_{I} + \nu\varepsilon_{II})}{(1-\nu^2)} = -60.4 \text{ MPa} \Leftarrow$$

$$\sigma_{II} = \frac{E(\varepsilon_{II} + \nu\varepsilon_{I})}{(1-\nu^2)} = -618.7 \text{ MPa} \Leftarrow$$

$\sigma_{III} = 0$

M21 a) Uniaxial loading

$$\varepsilon_{\cancel{x}z} = \frac{\sigma_z}{E} = \frac{100\times10^6}{3\times10^9} = 0.033 \Leftarrow$$

$$\varepsilon_y = \varepsilon_x = -\nu\varepsilon_z = -0.3 \times (0.033) = -0.01 \Leftarrow$$

b) Assume $\varepsilon_x = \varepsilon_y = 0$ $(E_{SiC} >> E_{epoxy})$

$$\begin{pmatrix} 0 \\ 0 \\ \varepsilon_z \end{pmatrix} = \begin{pmatrix} \frac{\sigma_x}{E} & -\frac{\nu\sigma_y}{E} & -\frac{\nu\sigma_z}{E} \\ -\frac{\nu\sigma_x}{E} & \frac{\sigma_y}{E} & -\frac{\nu\sigma_z}{E} \\ -\frac{\nu\sigma_x}{E} & -\frac{\nu\sigma_y}{E} & \frac{\sigma_z}{E} \end{pmatrix} \cancel{\begin{pmatrix} 0 \\ 0 \\ \varepsilon_z \end{pmatrix}}$$

by symmetry $\sigma_x = \sigma_y = \sigma_T$ $\quad \sigma_z = 100\ \text{MPa}$

$$0 = \frac{\sigma_T}{E}(1-\nu) - \frac{\nu\sigma_z}{E} \quad (1)$$

$$\varepsilon_z = \frac{-2\nu}{E}\sigma_T + \frac{\sigma_z}{E} \quad (2)$$

from (1) $$\sigma_T = \frac{\nu\sigma_z}{(1-\nu)} = \frac{0.3\times100\times10^6}{(1-0.3)}$$

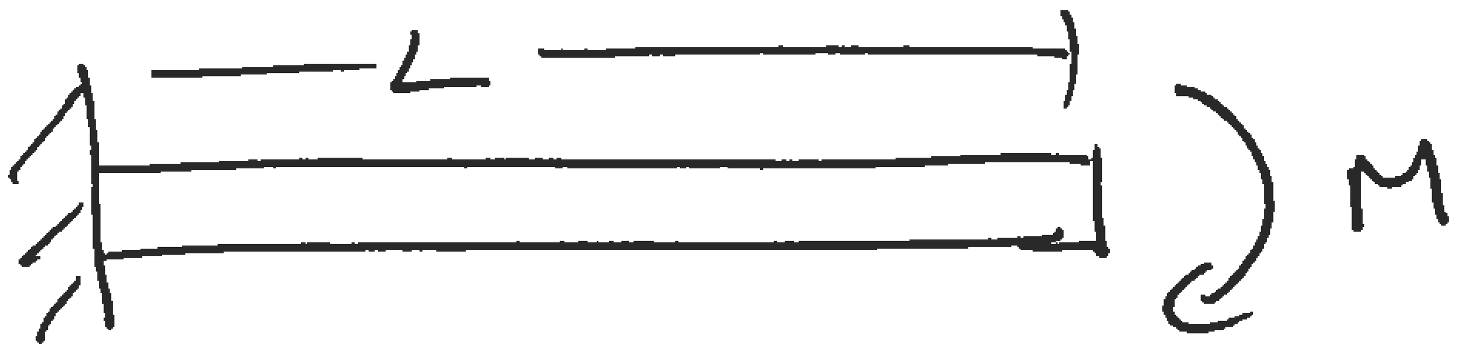
$$\sigma_T = 42.9\ \text{MPa}$$

$$\varepsilon_z = \frac{1}{E}\left(-2\nu\sigma_T + \sigma_z\right)$$

$$= \frac{1}{2\times10^9}\left(-2\times0.3\times42.9+100\right)\times10^6$$

$$= 0.0371 \Leftarrow .$$

(Note ~~ε_z~~ $\dfrac{\sigma_z}{E} = 0.05$ so transverse restraint makes epoxy appear stiffer)



M22 $\quad \delta = \dfrac{2ML^2}{\pi R^4 E}$ $\qquad$ Mass, $m = \rho \pi R^2 L$

R is the free variable, ∴ eliminate

$$R = \sqrt{\frac{m}{\rho \pi L}}$$

$$\Rightarrow \delta = \frac{2ML^2}{\pi E}\left(\frac{\rho \pi L}{m}\right)^2$$

$$\text{Mass} = \pi\left(\frac{2M}{\pi\delta}\right)^{\frac{1}{2}} \cdot \left(L^3\right)\left(\frac{\rho}{E^{\frac{1}{2}}}\right)$$

F $\qquad$ G $\qquad$ M

Minimize $\left(\dfrac{\rho}{E^{\frac{1}{2}}}\right)$, maximize $\dfrac{E^{\frac{1}{2}}}{\rho}$. ⇐

b)

	ρ (Mg/m³)	E (GPa)	$E^{1/2}/\rho$
Steel	7.9	203	1.8
Al	2.8	71	3.0
Ti	4.5	120	2.4
CFRP	1.5	230	10.11 ⇐
HDPE	0.96	1.1	1.1
Wood	0.6	12	5.8
SiC	3.0	410	6.8

Choose <u>CFRP</u>

c) choose material comparable in bending stiffness to bone - match $E^{1/2}/\rho$

18 GPa , $\rho = 1.55$

Draw line of constant $E^{1/2}/\rho$ on graph

possibilities

- low modulus GFRP, CFRP, KFRP laminates
  - Cement
  - Rock, stone
  - Ti alloys ⟸
  - ZrO_2

Choose Ti - these are used!