UNIFIED PROPULSION P6 SOLUTIONS WAITZ

a) $$I_{sp} = \frac{u_e}{g} = \frac{1}{g}\left[M_e\sqrt{\gamma R T_c\left(\frac{1}{1+\frac{\gamma-1}{2}M_e^2}\right)}\right]$$

NEED TO ITERATE TO FIND M_e FOR GIVEN $\frac{A_*}{A_e}$

$$\frac{A_e}{A^*} = \frac{0.01}{0.0006} = 16.67 = \frac{1}{M_e}\left[\frac{1+\frac{\gamma-1}{2}M_e^2}{\frac{\gamma+1}{2}}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \Longrightarrow M_e = 4.27$$

$$\boxed{\therefore\ I_{sp} = 271\ \text{s}} \qquad (u_e = 2654\ \text{m/s})$$

$$T = \dot{m}u_e + A_e(p_e - p_o)$$

p_o = ATMOSPHERIC PRESSURE AT LAUNCH

$$\dot{m} = \sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\frac{p_c}{\sqrt{RT_c}}A^* = 1.85\ \text{kg/s}$$

$$p_e = \frac{p_c}{\left[1+\frac{\gamma-1}{2}M_e^2\right]^{\gamma/\gamma-1}} = 20.0\ \text{kPa}$$

$$\boxed{\therefore\ T = 4102\,\text{N}\ \ (\text{AT LAUNCH})}$$

[= 4915 N if you neglect pressure term]

b) AT BURNOUT $$u_{bo} = g\left[I_{sp}\ln\left(\frac{m_{initial}}{m_{final}}\right) - t_{bo}\right]$$

$$h_{bo} = g\left[-t_{bo}I_{sp}\frac{\ln\left(\frac{m_i}{m_f}\right)}{\left(\frac{m_i}{m_f}-1\right)} + t_{bo}I_{sp} - \frac{1}{2}t_{bo}^2\right]$$

$$t_{bo} = m_{propellant}/\dot{m} = 35\text{kg}/2.14\ \text{kg/s} = 18.9\ \text{s}$$

SO $u_{bo} = 3010\ \text{m/s}$ $\quad h_{bo} = 22532\ \text{m}$

GLIDE PHASE $$\frac{1}{2}m_{final}u_{final}^2 = m_{final}\,g\,\Delta h$$

$$\Delta h = 462370\ \text{m}$$

TOTAL HEIGHT $= h_{bo} + \Delta h = \boxed{485\ \text{km}}$