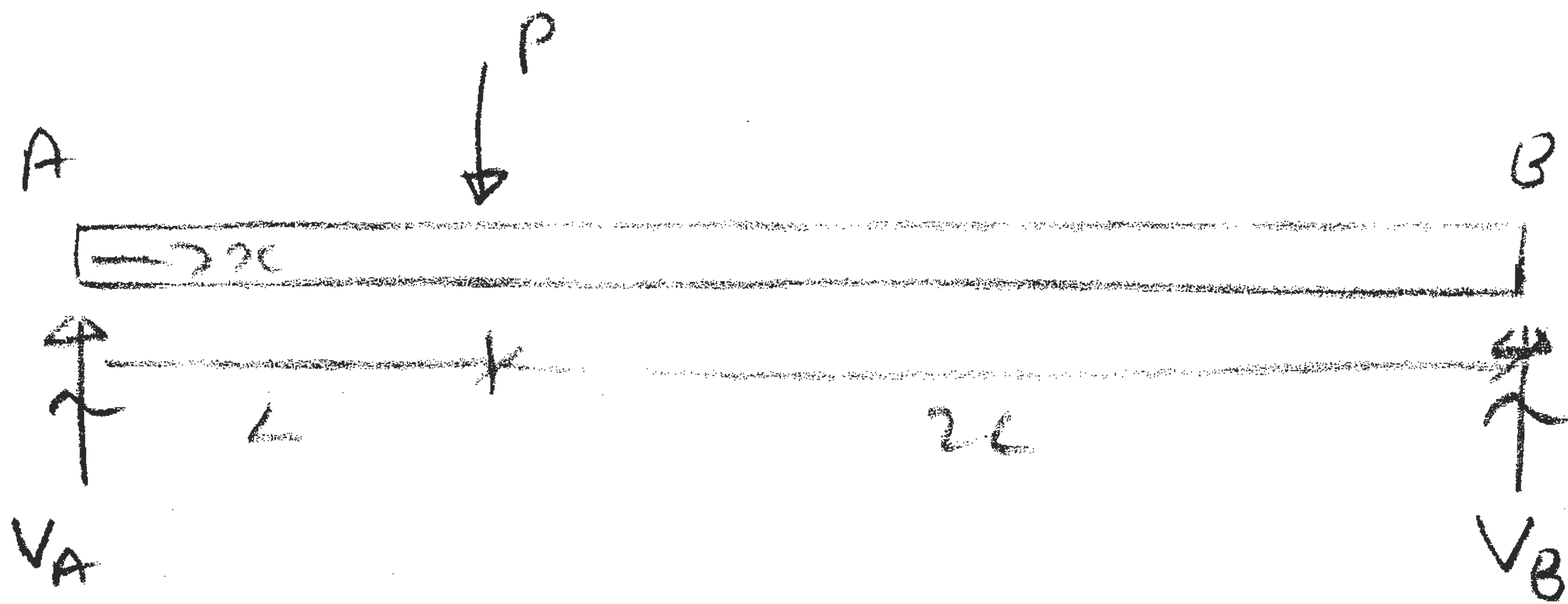
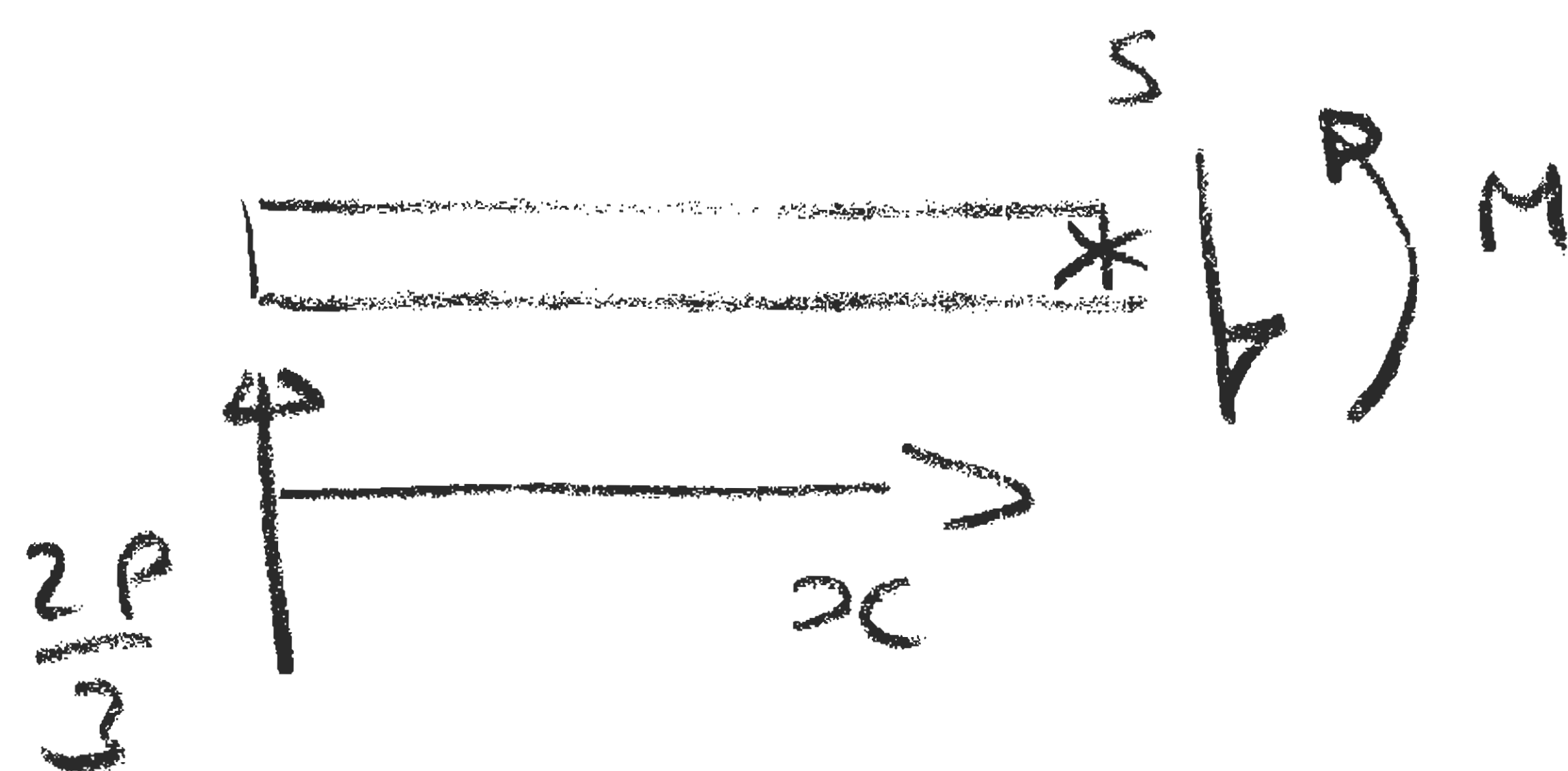


MS



$$\sum (M_A = 0) : 3LV_B - PL = 0 \quad ; \quad V_B = \frac{P}{3}, \quad V_A = \frac{2P}{3}$$

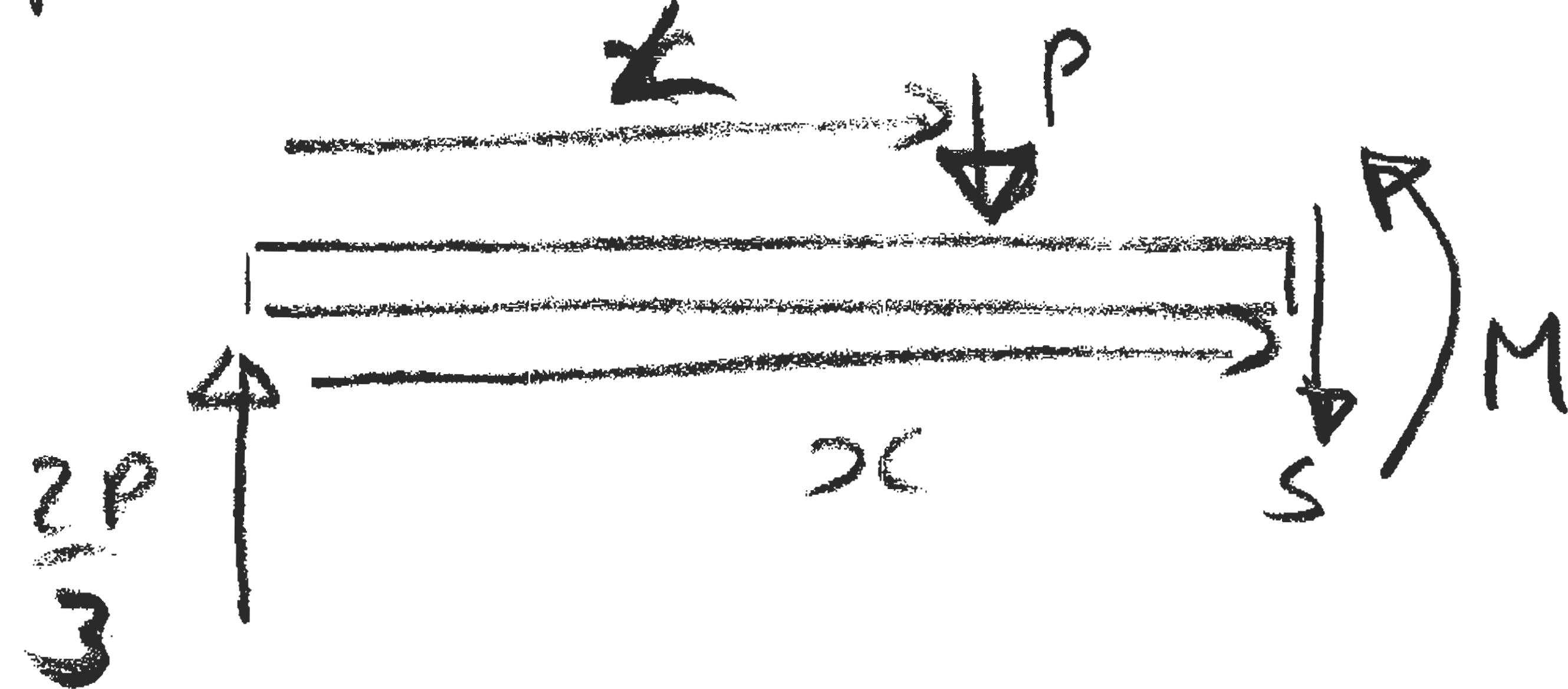
for  $0 < x < L$



$$\left( \sum M_x = 0 : M - \frac{2P}{3}x = 0 \right.$$

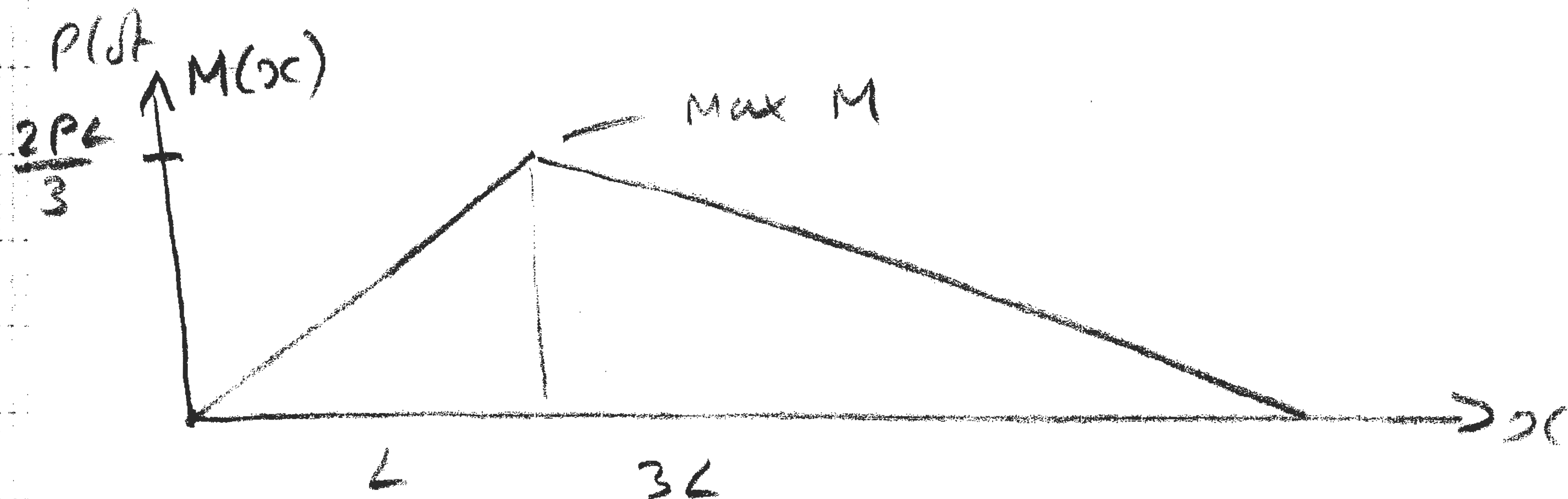
$$M = \frac{2Px}{3}$$

for  $L < x < 3L$



$$\left( \sum M_x = 0 : M + P(x-L) - \frac{2P}{3}x = 0 \right.$$

$$M = -\frac{P}{3}x + PL$$



deflection from

①

$$0 < x < L$$

$$EI \frac{d^2 w^1}{dx^2} = \frac{2Px}{3}$$

$$EI \frac{dw^1}{dx} = \frac{2Px^2}{6} + A$$

$$EI w^1 = \frac{2Px^3}{18} + Ax + B$$

$$EI \frac{d^2 w}{dx^2} = M$$

②

$$L < x < 3L$$

$$EI \frac{d^2 w^2}{dx^2} = -\frac{Px}{3} + PL$$

$$EI \frac{dw^2}{dx} = -\frac{Px^2}{6} + PLx + C$$

$$EI w^2 = -\frac{Px^3}{18} + \frac{PLx^2}{2} + Cx + D$$

Apply boundary conditions

$$\text{@ } x=0$$

$$w^1 = 0$$

$$x=3L \quad w^2 = 0$$

$$\Rightarrow B = 0,$$

$$-\frac{27PL^3}{18} + \frac{9PL^3}{2} + 3C \cancel{L} + D = 0$$

$$= 3PL^3 + 3C \cancel{L} + D = 0 \quad \text{①}$$

$$\text{@ } x=L$$

$$\frac{dw^1}{dx} = \frac{dw^2}{dx}$$

$$\Rightarrow \frac{2PL^2}{6} + A = -\frac{PL^2}{6} + PL^2 + C$$

$$A - C = \frac{PL^2}{2} \quad \text{②}$$

$$\textcircled{a} \quad x = L$$

$$w^1 = w^2$$

$$\frac{2PL^3}{18} + AL = -\frac{PL^3}{18} + \frac{PL^3}{2} + CL + D$$

$$(A - C)L = \frac{PL^3}{3} + D \quad \textcircled{3}$$

$$\text{Substitute for } A - C = \frac{PL^2}{2} \quad \textcircled{2}$$

$$\Rightarrow \frac{PL^3}{2} = \frac{PL^3}{3} + D \Rightarrow D = \frac{PL^3}{6} \Leftarrow$$

$$\text{Substitute in } \textcircled{1} \quad 3PL^2 + 3Cx + \frac{PL^2}{6} = 0$$

$$C = \frac{1}{3} \left( -3PL^2 - \frac{PL^2}{6} \right) = -\frac{19}{18} PL^2 \Leftarrow$$

$$\therefore A = \frac{PL^2}{2} + \frac{19}{18} PL^2 = \frac{14}{9} PL^2 \Leftarrow$$

$\therefore$  for  $0 < x < L$

for  $L < x < 3L$

$$w = \frac{1}{EI} \left( \frac{2Px^3}{18} + \frac{14PL^2}{9} x \right) \quad w = \frac{1}{EI} \left( -\frac{Px^2}{2} + \dots \right)$$

for  $L < x < 3L$

$$w = \frac{1}{EI} \left( -\frac{Px^3}{18} + \frac{PLx^2}{2} - \frac{19PL^2x}{18} + \frac{PL^3}{6} \right) \quad \checkmark$$

Max deflection when  $\frac{dw}{dx} = 0$

between  $0 < x < L$ ?

$$\frac{dw}{dx} = \frac{1}{EI} \left( \frac{6Px^2}{18} + \frac{28PL^2x}{9} \right) = 0 \quad \text{Not in } 0 < x < L$$

between  $L < x < 3L$  or  $1 < \frac{x}{L} < 3$

$$\frac{dw}{dx} = \frac{PL^2}{EI} \left( -\frac{3x^2}{18L^2} + \frac{2x}{L} - \frac{19}{18} \right) = 0$$

$$\text{for } \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \frac{-1 \pm \sqrt{1 - 4 \times \frac{1}{6} \times \frac{19}{18}}}{-\frac{2}{6}}$$

Root between  $L$  and  $3L = x = 1.37L$

$$= \delta = \frac{PL^2}{EI} \left( \frac{-1}{5} \right) = -\frac{PL^2}{5EI} \quad \ominus$$

Note this does not occur at  $x = L$  where  
 $M = \text{Maximum}$ .