**Massachusetts Institute of Technology**
**Department of Aeronautics and Astronautics**
**Cambridge, MA 02139**

## 16.01/16.02 Unified Engineering I, II
## Fall 2003

### Problem Set 11

Name: _____

Due Date: 11/18/03

| | Time Spent (min) |
|---|---|
| **F10** | |
| **F11** | |
| **M11** | |
| **M12** | |
| **Study Time** | |

Announcements:

F10.

a) Determine the streamline shapes of the following 2-D velocity field (closely related to HW problem F6).

$$u = -y \qquad\qquad v = x$$

b) Evaluate $Du/Dt$ and $Dv/Dt$, and determine the pressure gradient $\nabla p$. Assume the density $\rho$ is constant (low speed flow).

c) Using your result from b), determine the pressure field $p(x, y)$ to within an additive constant.


F11.

a) The flow inside a boundary layer near a wall has a simple linear velocity distribution $u = Cy$, $v = 0$. Determine the vorticity and strain rate of the fluid.

b) A fluid element initially has a square shape. Determine and sketch its shape a short time later. Again using a sketch, describe how its shape relates to rotation and shearing element motions.
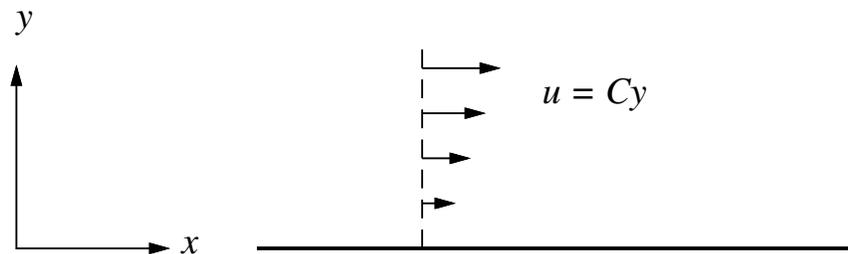
**Problem M 11**

For the truss structure and the loading shown below, you are to calculate the cross-sectional area of the bars to achieve both of the following design objectives:

1) The maximum stress in any of the members is not to exceed 200 MPa

2) The vertical deflection of point D is not to exceed 25 mm
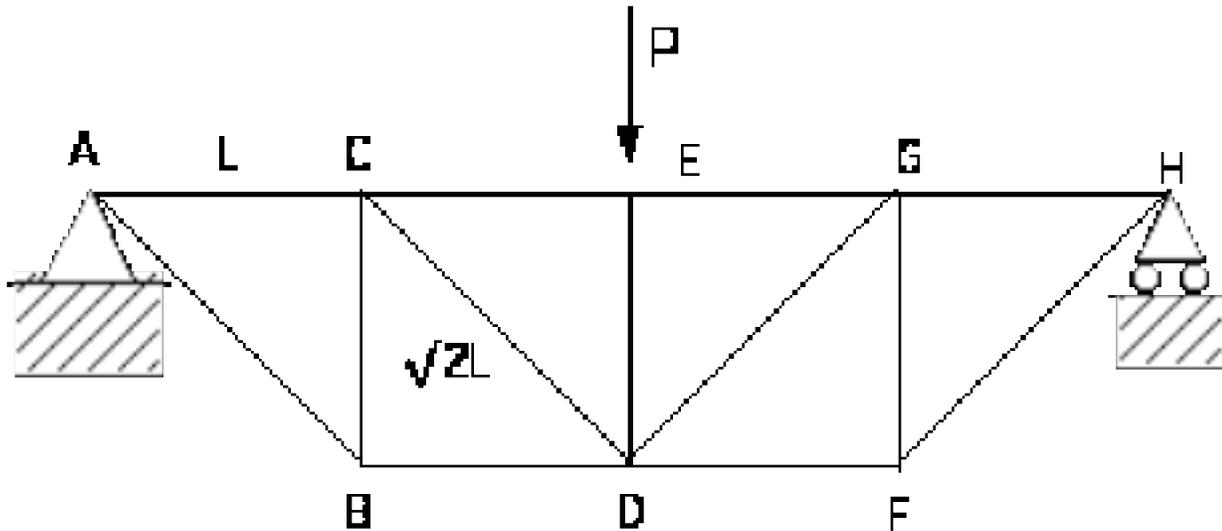


Estimate the mass of the resulting structure. Does this seem reasonable?

All of the bars are to be made of tubes of Aluminum Alloy 7075, Young's modulus 70 GPa, density 2700 kg/m³ . All tubes will have the same cross-sectional area.

List all of the assumptions you are making in your calculation. Suggest a means of reducing the mass of the structure further.
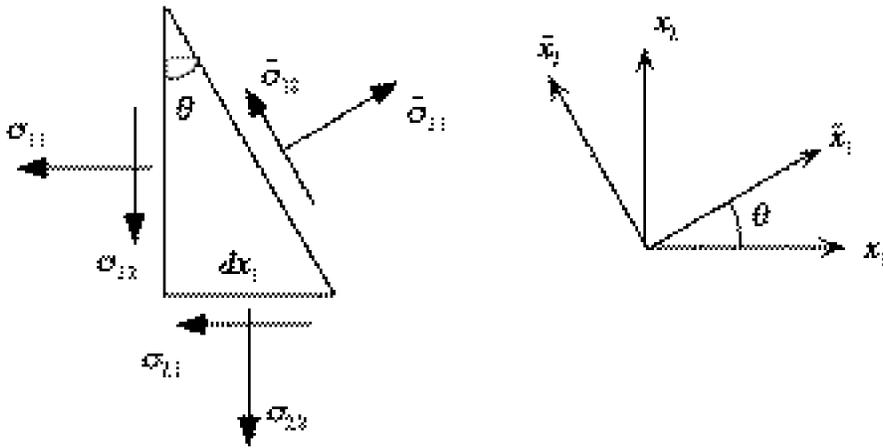
**Problem M12**  (Materials and Structures)

The figure below shows a triangular element in a two dimensional plane.  The element is defined by an angle θ and the length of the opposite side $dx_1$.    The element is of uniform thickness,  $dx_3$.  The element is acted on by a state of stress in the plane, $\sigma_{11}$, $\sigma_{22}$ and $\sigma_{12}$.  Stresses $\tilde{\sigma}_{11}$ and $\tilde{\sigma}_{12}$ y corresponding to a rotated axis system, $\tilde{x}_\beta$ .  By considering equilibrium of the forces acting on the triangular element drawn below show that:

$$\tilde{\sigma}_{11} = \cos^2\theta\,\sigma_{11} + \sin^2\theta\,\sigma_{22} + 2\cos\theta\sin\theta\,\sigma_{12} \tag{1}$$

and

$$\tilde{\sigma}_{12} = -\cos\theta\sin\theta\,\sigma_{11} + \cos\theta\sin\theta\,\sigma_{22} + \left(\cos^2\theta - \sin^2\theta\right)\sigma_{12} \tag{2}$$



Find the values of θ that produce the maximum and minimum values of $\tilde{\sigma}_{11}$, what are the corresponding values of $\tilde{\sigma}_{12}$?.  Do not try to distinguish between the maximum and minimum values.

Note:  This is a "plane stress" problem, i.e. stresses only act in the plane of the drawing ($\sigma_{33}=\sigma_{13}=\sigma_{23}=0$).  This problem is at the heart of transforming stress, take the time to make sure that you understand the procedure.