UE Fluids | F16 Solution | Fall '03

a) Given: $\phi_1(x,y)$, $\phi_2(x,y)$, $p_1(x,y)$, $p_2(x,y)$

$\nabla^2\phi_1 = 0$, $\nabla^2\phi_2 = 0$ (satisfy mass conservation)

Define: $\phi_3 = \phi_1 + \phi_2$

Determine p_3

$$\Rightarrow p_3 = p_0 - \tfrac{1}{2}\rho|\nabla\phi_3|^2 = p_0 - \tfrac{1}{2}\rho(\nabla\phi_3 \cdot \nabla\phi_3)$$

$$= p_0 - \tfrac{1}{2}\rho\left[(\nabla\phi_1 + \nabla\phi_2)\cdot(\nabla\phi_1 + \nabla\phi_2)\right]$$

$$\boxed{p_3 = p_0 - \tfrac{1}{2}\rho\left[|\nabla\phi_1|^2 + |\nabla\phi_2|^2 + 2\nabla\phi_1 \cdot \nabla\phi_2\right]}$$

Note: $p_3 \neq p_1 + p_2$!

b) Given $\phi_4 = \partial\phi_1/\partial x$

Is it physically realizable?

Test: $\nabla^2\phi_4 \overset{?}{=} 0$

$$\nabla^2(\partial\phi_1/\partial x) \overset{?}{=} 0$$

$$\boxed{\frac{\partial}{\partial x}(\nabla^2\phi_1) \overset{\checkmark}{=} 0}$$

$\frac{\partial()}{\partial x}$ and $\nabla^2()$ commute.

Since $\nabla^2\phi_1 = 0$ as given

Example: $\phi_1 = \frac{\Lambda}{2\pi}\ln\sqrt{x^2+y^2} = \frac{\Lambda}{2\pi}\ln r$ source

$\phi_4 = \frac{\partial\phi_1}{\partial x} = \frac{\Lambda}{2\pi}\frac{x}{x^2+y^2} = \frac{\Lambda}{2\pi}\frac{\cos\theta}{r}$ doublet!